Optimization (Section 4.6)

Intro

Today is the most applied we get this semester. We will solve real life problems involving quantities that people actually get paid to optimize.

- Costs (minimize)
- Revenue and Profits (maximize)
- Time spent traveling (minimize)
- Materials for construction (minimize)
- Floor area for housing (maximize)

Example

We want to draw a rectangle inside the top half of an ellipse that has the largest area possible. The ellipse has equation

$$1 = \frac{x^2}{4^2} + \frac{y^2}{2.5^2}.$$

How big should we make the base so that the area is maximized?

Basic principles

- Draw a picture of the scenario.
- Label the relevant variables in your picture
- Identify what quantity you are optimizing. Write down that equation
- Write down a second equation from the information given (the constraining equation).
- Solve the second equation for a variable. Substitute it into the first equation.
- The first equation should now have only one variable. Minimize/maximize it.
- Make sure to answer the question!

Example

Find two numbers a and b such that their difference is 100 and their product is as large as possible

Example

Find the two positive numbers a and b such that their product equals 100 and whose sum is minimized.

Example

Find two numbers a and b such that the sum of a and twice b is 400 and such that their product is maximized.

Example

A cardboard box with a square base and an open top has a volume of $10\,m^3$. Find the dimensions of the box minimizing the amount of cardboard needed to build the box. How much cardboard do we need?

Example

We have 300 ft^2 of plywood available to build a box with a square base and open on the top. Determine the dimensions which will maximize the volume of the box. How much volume will we get?