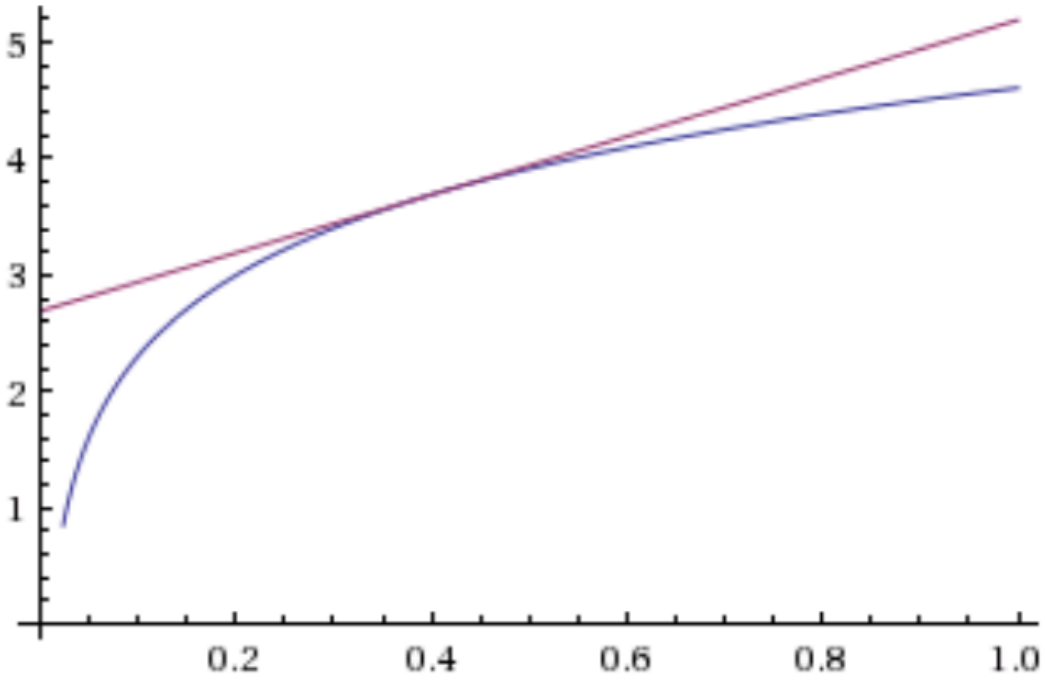
Linear Approximations and Differentials (Section 3.9)

Intro

Linear approximations are one more way of applying derivatives to real world problems.

Linear approximation

A **linear approximation** is another name for a tangent line. The tangent line at $x = a$ is a close estimate to the graph of the function, as long as we are close to a.



Linear approximation

As long as a graph is differentiable at a, if we zoom in close enough it looks like a line.

Example

Find the linear approximation of $f(x)$ at $x = -1$.

$$f(x) = x^4 + 2x^2$$

Example

Use a linearization to estimate the following number

$$(32.06)^{4/5}$$

Example

Use a linearization to estimate the following number

$$\sin(0.99\pi)$$

Differential

The **differential** dy is defined as

$$dy = f'(x)dx$$

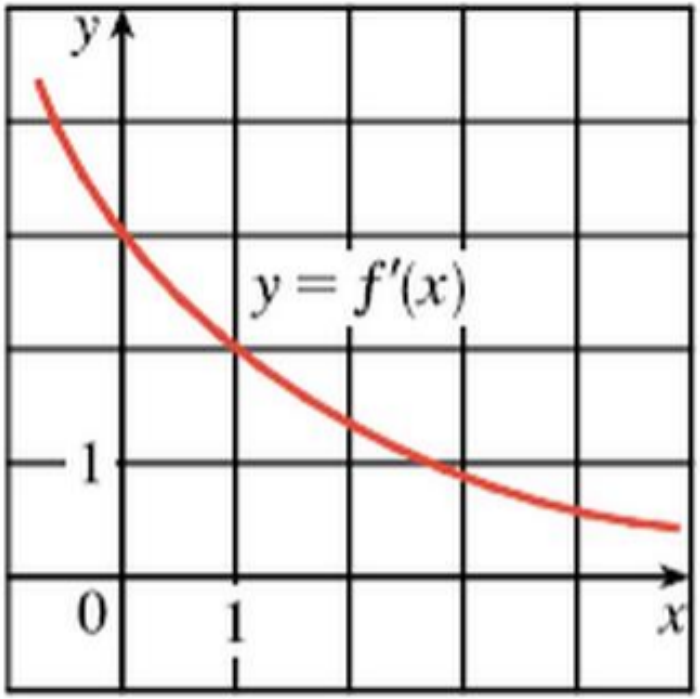
To get a more rigorous definition, take Math 662.

Example

Find the differential dy for $y = \dfrac{u+1}{u-1}$.

Example

Suppose that $f(1) = -2$ and that the graph of $f'(x)$ is



Estimate $f(0.99)$ and $f(1.01)$.

Example

Find the linear approximation for $\sqrt{16-x}$. Use this to approximate $\sqrt{15.9}$ and $\sqrt{15.99}$